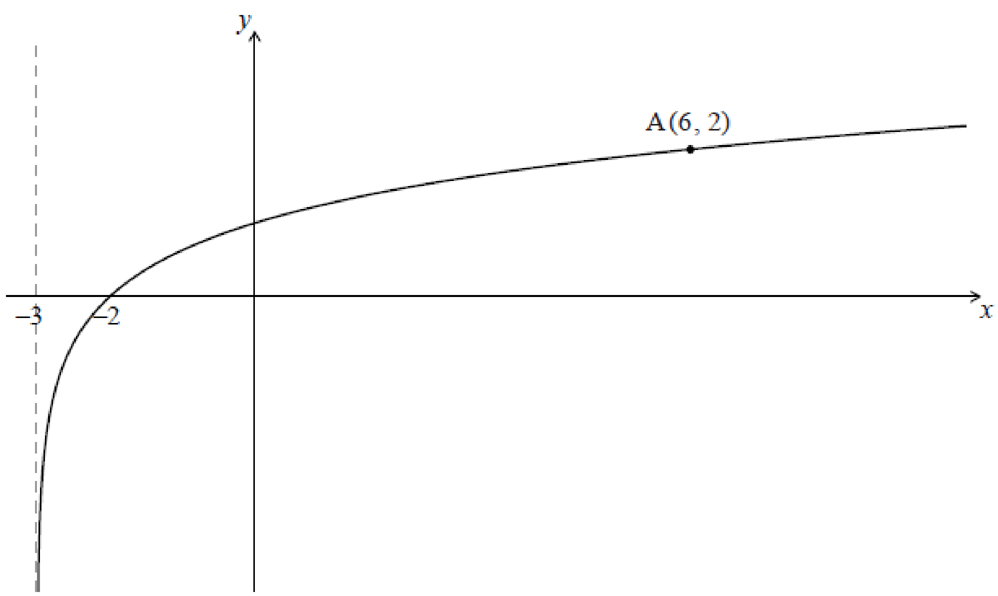
SL Paper 1

Let $f(x) = (x-5)^3$, for $x \in \mathbb{R}$.

a. Find $f^{-1}(x)$. [3]

b. Let g be a function so that $(f \circ g)(x) = 8x^6$. Find $g(x)$. [3]

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of *f* is shown below.



The graph passes through A(6, 2) , has an *x*-intercept at (−2, 0) and has an asymptote at $x = -3$.

a. Find *p* . [4]

b. The graph of *f* is reflected in the line $y = x$ to give the graph of *g* . [5]

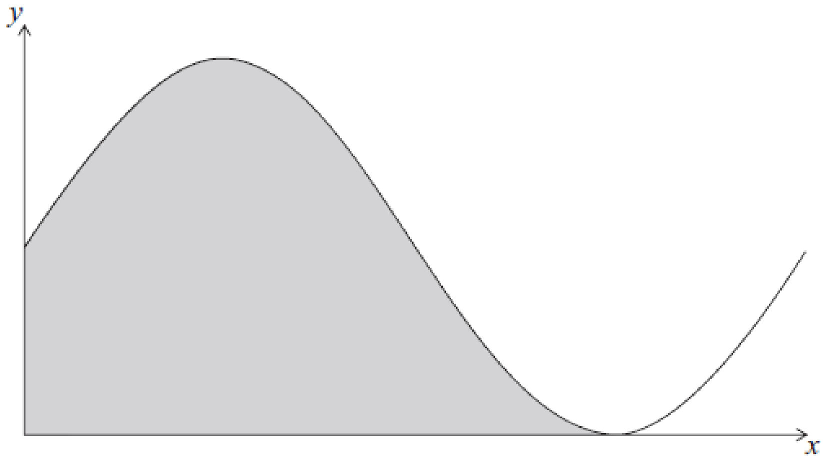
(i) Write down the *y*-intercept of the graph of *g* .

(ii) Sketch the graph of *g* , noting clearly any asymptotes and the image of A.

c. The graph of f is reflected in the line $y = x$ to give the graph of g . [4]

Find $g(x)$.

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f , the x-axis, and the y-axis.

a(i) and (ii). Solve for $0 \le x < 2\pi$ [5]

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

b. Write down the exact value of the x-intercept of f , for $0 \le x < 2\pi$. [1]

c. The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6]

d. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [2]

Give a full geometric description of this transformation.

e. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [3]

Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p.

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

a. The graph of g can be obtained from the graph of f using two transformations. [2]

Give a full geometric description of each of the two transformations.

b. The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h. [4]

The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h.

Find the coordinates of P.

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

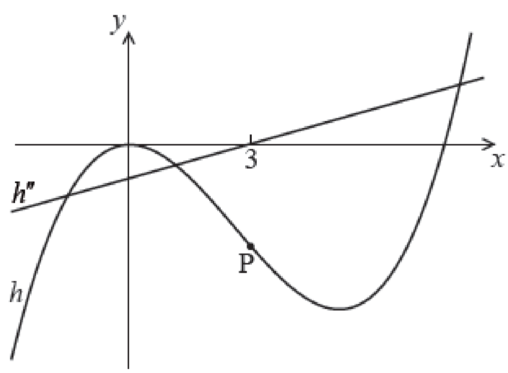
a. Show that the two zeros are 3 and -6. [2]

b. Find the value of q and of r. [4]

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	–14	–18
$f'(x)$	1	1
$g'(x)$	–5	–3
$h''(x)$	–6	0

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

Given that $h(x) = f(x) \times g(x)$,

a. Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. [3]

b. Explain why P is a point of inflexion. [2]

c. find the y-coordinate of P. [2]

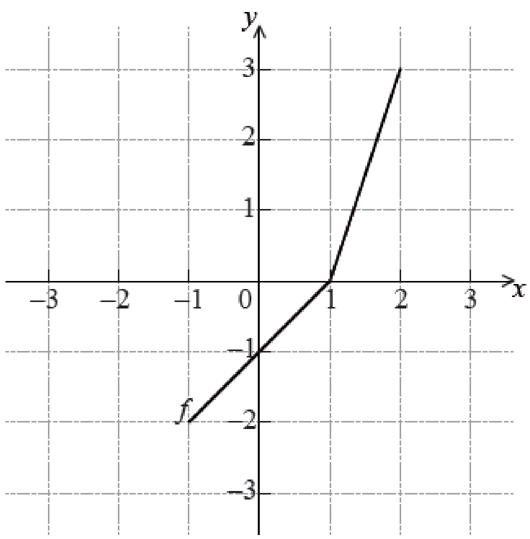
d. find the equation of the normal to the graph of h at P. [7]

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

a. Find $f^{-1}(x)$. [3]

b. Find $(f \circ g)(1)$. [3]

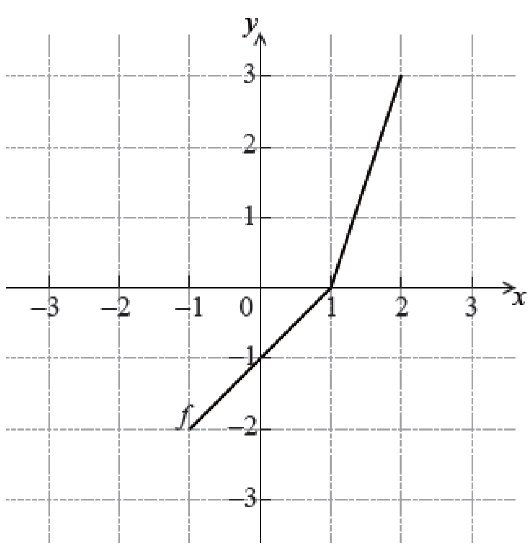
The diagram below shows the graph of a function f , for $-1 \leq x \leq 2$.



a.i. Write down the value of $f(2)$. [1]

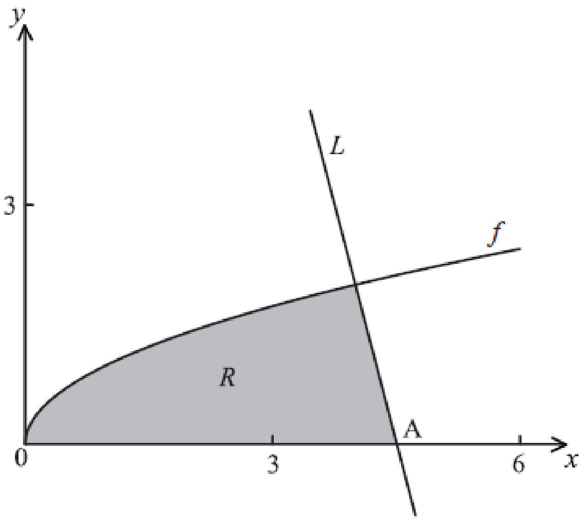
a.ii. Write down the value of $f^{-1}(-1)$. [2]

b. Sketch the graph of f^{-1} on the grid below. [3]



Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2) .

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L .



a. Show that the equation of L is $y = -4x + 18$. [4]

b. Point A is the x-intercept of L . Find the x-coordinate of A. [2]

c. Find an expression for the area of R . [3]

d. The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π . [8]

Let $f(x) = 3(x+1)^2 - 12$.

a. Show that $f(x) = 3x^2 + 6x - 9$. [2]

b(i), (ii) and (iii). For the graph of f [7]

(i) write down the coordinates of the vertex;

(ii) write down the y-intercept;

(iii) find both x-intercepts.

c. **Hence** sketch the graph of f . [3]

d. Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the following two transformations [3]

a stretch of scale factor t in the y-direction,

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t .

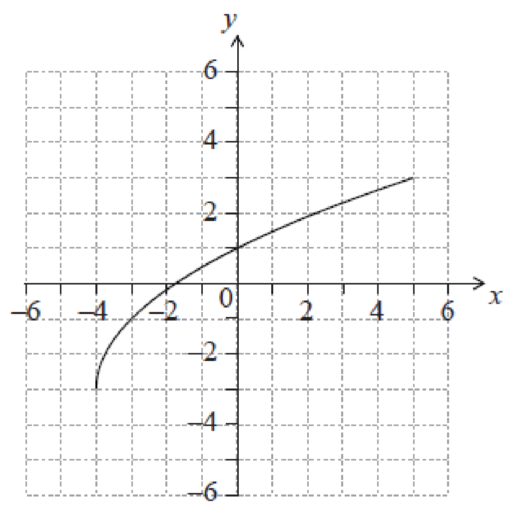
Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

a. Find $(g \circ f)(x)$. [2]

b. Write down $g^{-1}(x)$. [1]

c. Find $(f \circ g^{-1})(5)$. [2]

The following diagram shows the graph of $y = f(x)$, for $-4 \leq x \leq 5$.



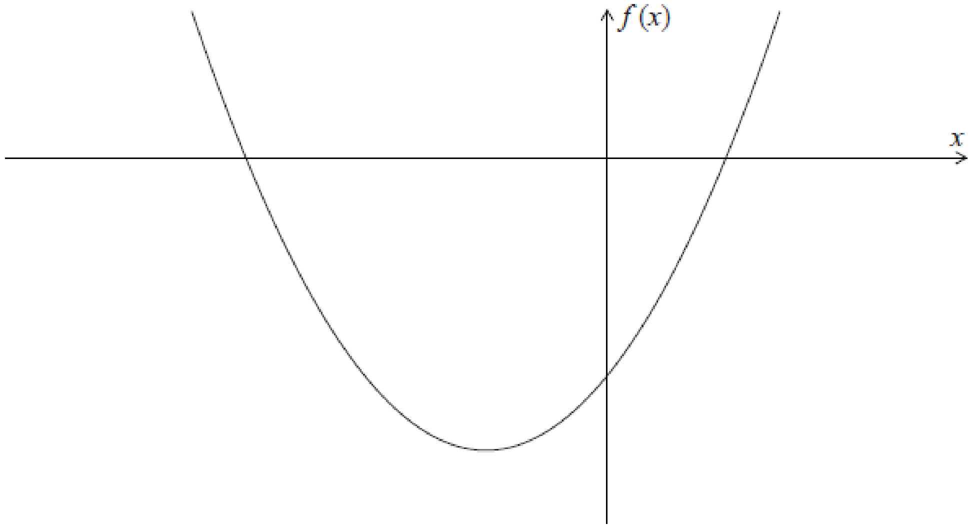
a(i).Write down the value of $f(-3)$. [1]

a(ii)Write down the value of $f^{-1}(1)$. [1]

b. Find the domain of f^{-1}. [2]

c. On the grid above, sketch the graph of f^{-1}. [3]

The diagram below shows part of the graph of $f(x) = (x - 1)(x + 3)$.



. (a) Write down the x-intercepts of the graph of f . [6]

(b) Find the coordinates of the vertex of the graph of f .

a. Write down the x-intercepts of the graph of f . [2]

b. Find the coordinates of the vertex of the graph of f . [4]

Let $f(x) = \sqrt{x+2}$ for $x \geq 2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

a. Write down $f(14)$. [1]

b. Find $(g \circ f)$ (14). [2]

c. Find $g^{-1}(x)$. [3]

Let $f(x) = 8x + 3$ and $g(x) = 4x$, for $x \in \mathbb{R}$.

a. Write down $g(2)$. [1]

b. Find $(f \circ g)(x)$. [2]

c. Find $f^{-1}(x)$. [2]

Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

a. Find $f^{-1}(x)$. [3]

b. Find $(f \circ g)(1)$. [3]

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

a(i). Write down the **value** of the discriminant. [2]

a(ii) Hence, show that $p = 3$. [1]

b. The graph of f has its vertex on the x-axis. [4]

Find the coordinates of the vertex of the graph of f.

c. The graph of f has its vertex on the x-axis. [1]

Write down the solution of $f(x) = 0$.

d(i). The graph of f has its vertex on the x-axis. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of a.

d(ii) The graph of f has its vertex on the x-axis. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of h.

d(iii) The graph of f has its vertex on the x-axis. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of k.

e. The graph of f has its vertex on the x-axis. [4]

The graph of a function g is obtained from the graph of f by a reflection of f in the x-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

Find g, giving your answer in the form $g(x) = Ax^2 + Bx + C$.

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$

Let $g(x) = f\left(x^2 + 1\right)$ and P be the point on the graph of g where $x = 1$.

a.i. Write down $f'(2)$. [2]

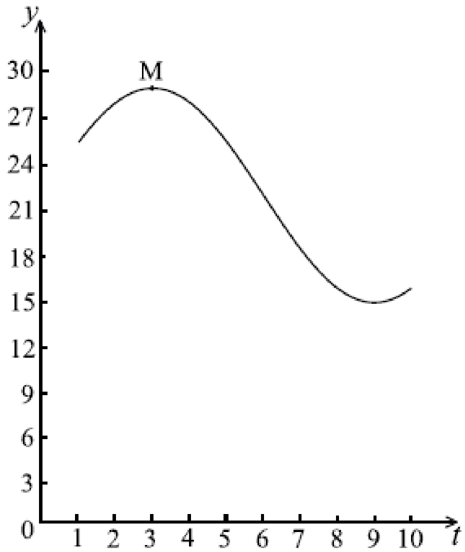
a.ii. Find $f(2)$. [2]

b. Show that the graph of g has a gradient of 6 at P. [5]

c. Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. [7]

Find the y-coordinate of Q.

Let $f(t) = a\cos b(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.

When $t = 9$, there is a minimum value of 15.

a(i), (ii), (iii) and (iv). (i) Find the value of a. [7]

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

b. The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$. [2]

Let M' be the image of M under P. Find the coordinates of M' .

c. The graph of g is the image of the graph of f under P. [4]

Find $g(t)$ in the form $g(t) = 7\cos B(t - c) + D$.

d. The graph of g is the image of the graph of f under P. [3]

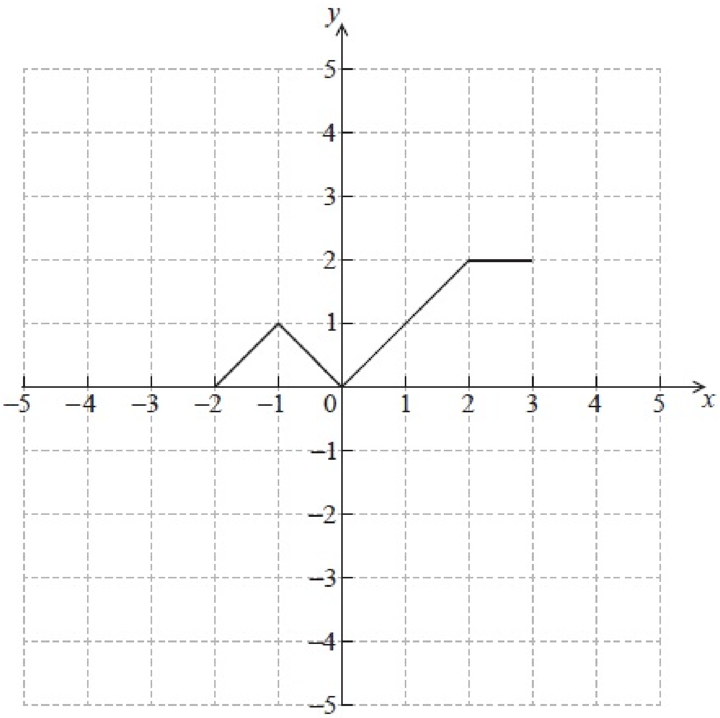
Give a full geometric description of the transformation that maps the graph of g to the graph of f .

Let $f(x) = a(x - h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

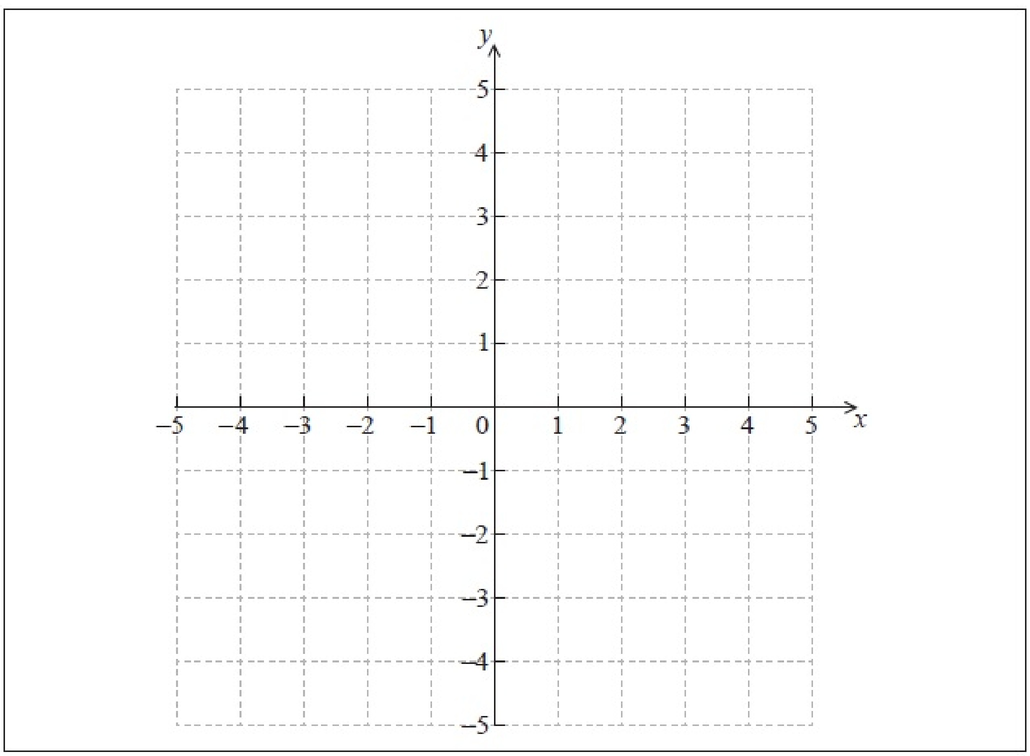
a. Write down the value of h and of k. [2]

b. Find the value of a. [3]

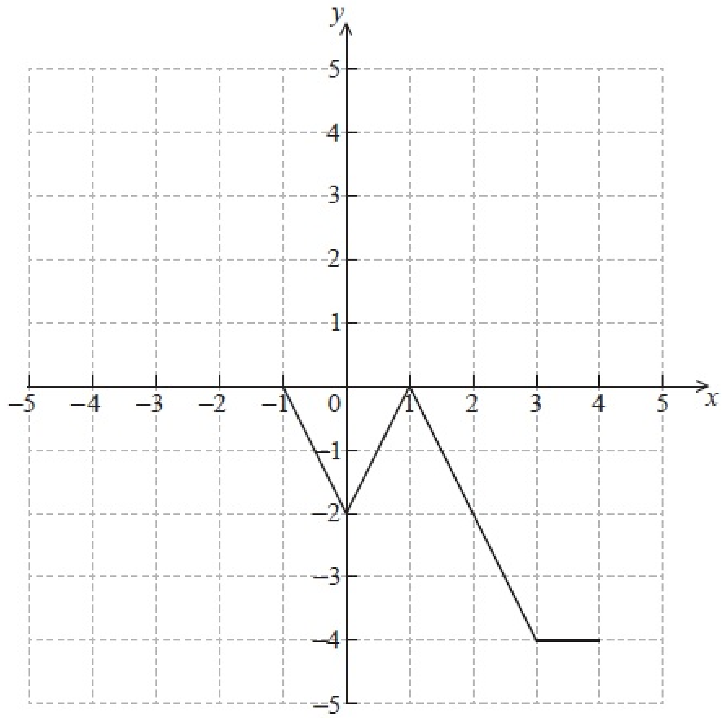
The diagram below shows the graph of a function $f(x)$, for $-2 \leq x \leq 3$.



a. Sketch the graph of $f(-x)$ on the grid below. [2]



b. The graph of *f* is transformed to obtain the graph of g . The graph of g is shown below. [4]



The function g can be written in the form $g(x) = af(x+b)$. Write down the value of a and of b .

Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

a. Find $(f \circ g)(x)$. [2]

b. The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h . [3]

Find the coordinates of the vertex of the graph of h .

c. The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h . [2]

Show that $h(x) = x^2 - 8x + 19$.

d. The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h . [5]

The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P.

Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m.

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

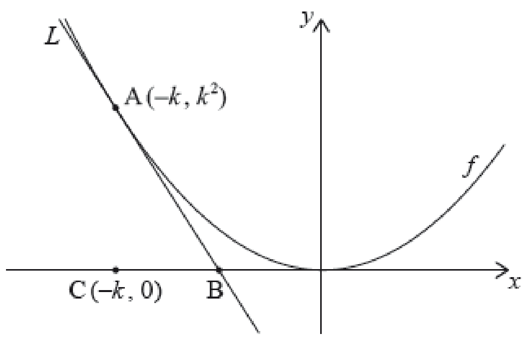
a. (i) Find $g(0)$. [5]

(ii) Find $(f \circ g)(0)$.

b. Find $f^{-1}(x)$. [3]

Let $f(x) = x^2$. The following diagram shows part of the graph of f.

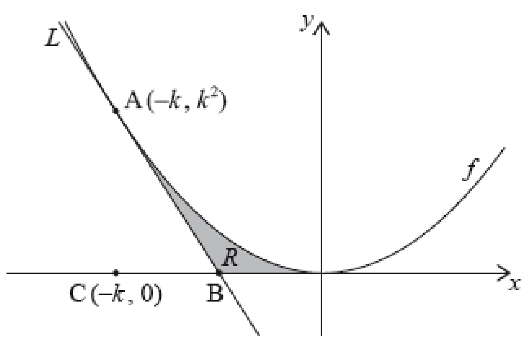
diagram not to scale



The line L is the tangent to the graph of f at the point $\mathrm{A}(-k,\ k^2)$, and intersects the x-axis at point B. The point C is $(-k,\ 0)$.

The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

diagram not to scale



a.i. Write down $f'(x)$. [1]

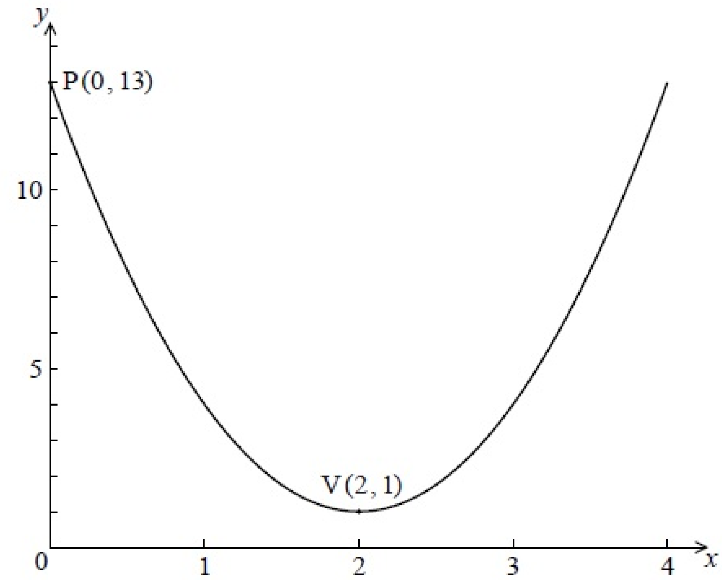
a.ii. Find the gradient of L. [2]

b. Show that the x-coordinate of B is $-\frac{k}{2}$. [5]

c. Find the area of triangle ABC, giving your answer in terms of k. [2]

d. Given that the area of triangle ABC is p times the area of R, find the value of p. [7]

The following diagram shows the graph of a quadratic function f, for $0 \leq x \leq 4$.



The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

a(i) and (ii). The function can be written in the form $f(x) = a(x - h)^2 + k$. [4]

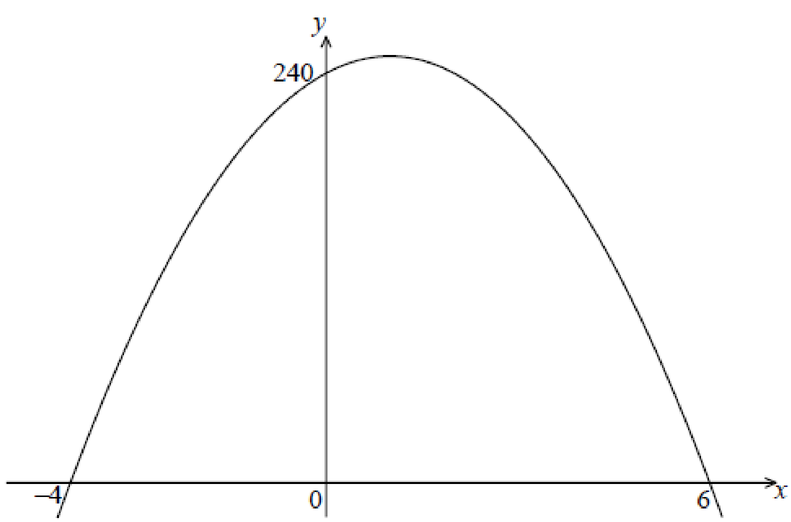
(i) Write down the value of h and of k .

(ii) Show that $a = 3$.

b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. [3]

c. Calculate the area enclosed by the graph of f , the x-axis, and the lines $x = 2$ and $x = 4$. [8]

The following diagram shows part of the graph of a quadratic function f .



The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

a. Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$. [2]

b. Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. [4]

c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2]

d(i) And (ii). A particle moves along a straight line so that its velocity, v ms^{-1} , at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$. [7]

(i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

Let $f(x) = x^2 - 4x + 5$.

The function can also be expressed in the form $f(x) = (x-h)^2 + k$.

a. Find the equation of the axis of symmetry of the graph of f. [2]

b. (i) Write down the value of h. [4]

(ii) Find the value of k.

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

a. Find $f\left(\frac{\pi}{2}\right)$. [2]

b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$. [2]

c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. [3]

Let $f(x) = \ln(x+5) + \ln 2$, for $x > -5$.

a. Find $f^{-1}(x)$. [4]

b. Let $g(x) = \mathrm{e}^x$. [3]

Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$.

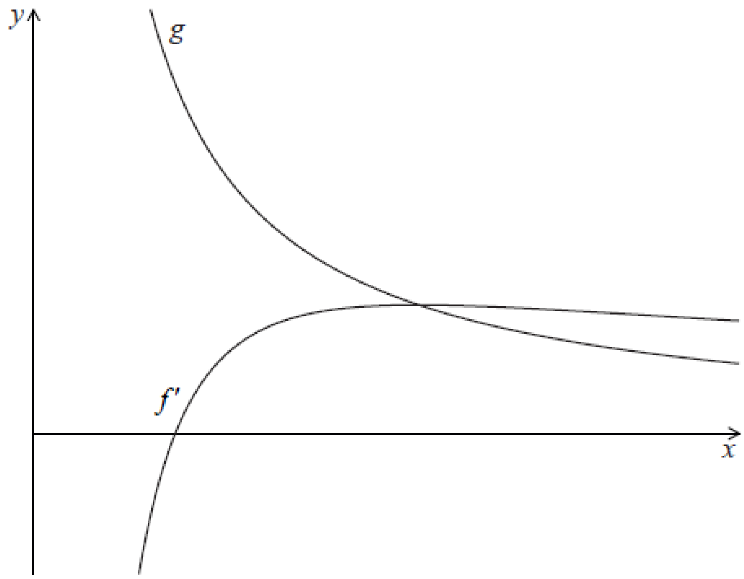
Let $f(x) = k\log_2 x$.

a. Given that $f^{-1}(1) = 8$, find the value of k . [3]

b. Find $f^{-1}\left(\frac{2}{3}\right)$. [4]

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

a. Show that $f'(x) = \frac{\ln x}{x}$. [2]

b. There is a minimum on the graph of f. Find the x-coordinate of this minimum. [3]

c. Write down the value of p. [2]

d. The graph of g intersects the graph of f' when $x = q$. [3]

Find the value of q.

e. The graph of g intersects the graph of f' when $x = q$. [5]

Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$.
Show that the area of R is $\frac{1}{2}$.

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

a. Find $(g \circ f)(x)$. [2]

b. Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

Let $f(x) = log_3\sqrt{x}$, for $x > 0$.

a. Show that $f^{-1}(x) = 3^{2x}$. [2]

b. Write down the range of f^{-1} . [1]

c. Let $g(x) = \log_3 x$, for $x > 0$. [4]

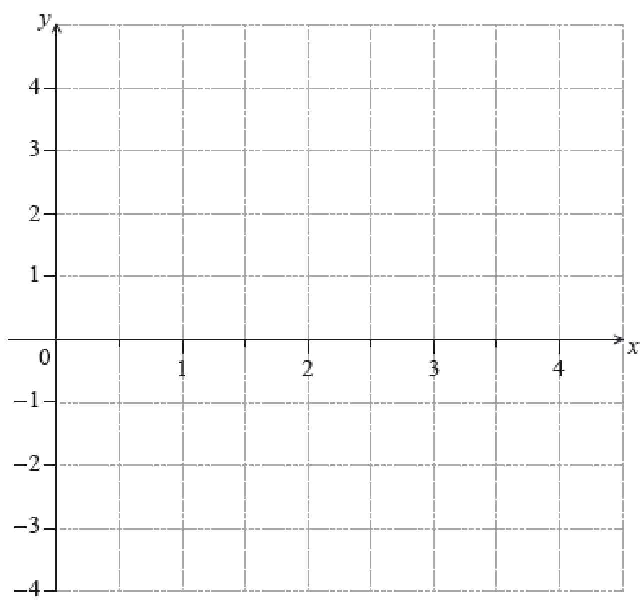
Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

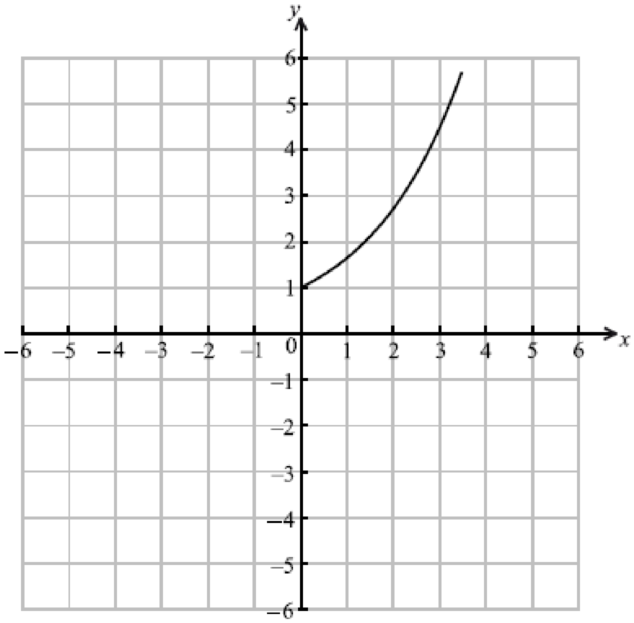
a. (i) Write down the amplitude of f. [3]

(ii) Find the period of f.

b. On the following grid sketch the graph of f. [4]



Let f be the function given by $f(x) = e^{0.5x}$, $0 \leq x \leq 3.5$. The diagram shows the graph of f .



a. On the same diagram, sketch the graph of f^{-1} . [3]

b. Write down the range of f^{-1} . [1]

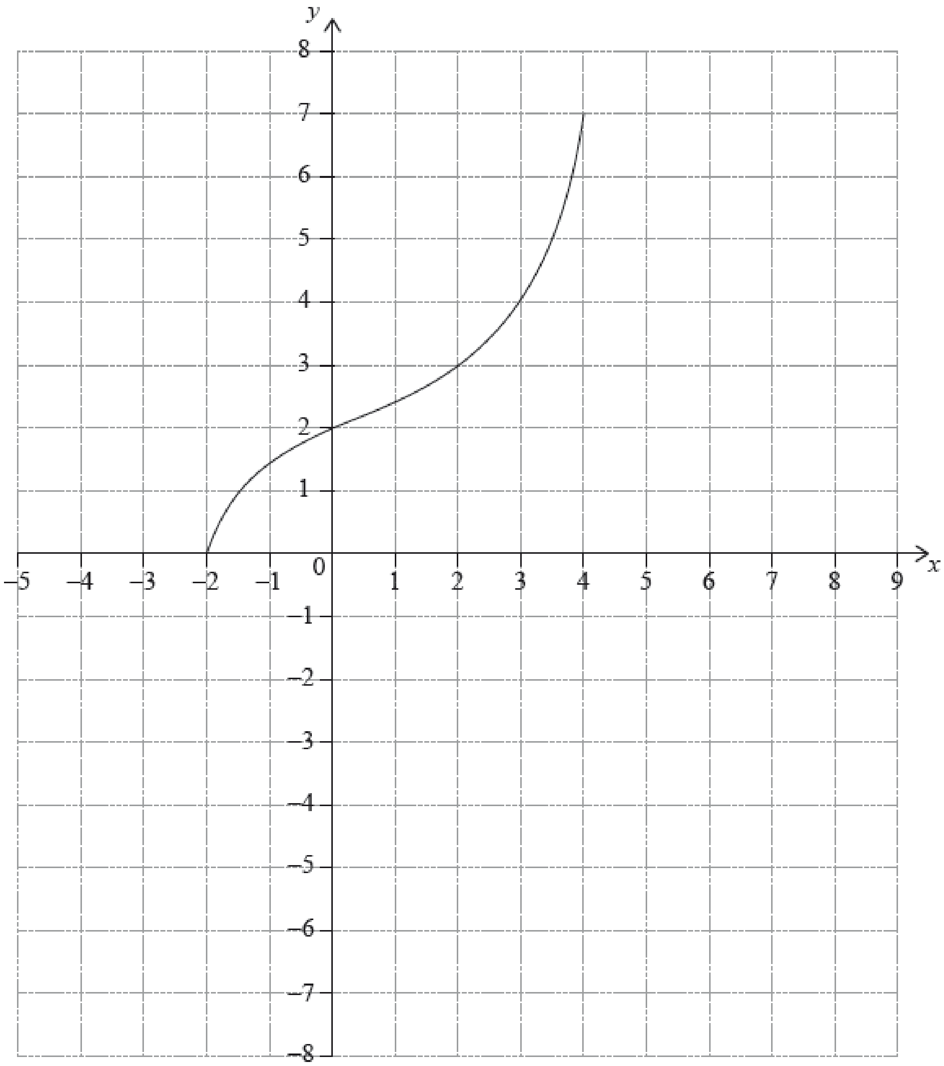
c. Find $f^{-1}(x)$. [3]

Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

a. Find $f^{-1}(x)$. [2]

b. Find $(f \circ g)(7)$. [3]

The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

a. Write down the range of f. [1]

b.i. Write down $f(2)$; [1]

b.ii. Write down $f^{-1}(2)$. [1]

c. On the grid, sketch the graph of f^{-1}. [3]

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

a. Find the values of k such that $f(x) = 0$ has two equal roots. [4]

b. Each value of k is equally likely for $-5 \le k \le 5$. Find the probability that $f(x) = 0$ has no roots. [4]

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

Let $h(x) = \frac{5}{x+2}$, for $x \geqslant 0$. The graph of h has a horizontal asymptote at $y = 0$.

a. Find $f^{-1}(x)$. [2]

b. Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$. [2]

c(i). Find the y-intercept of the graph of h. [2]

c(ii). Hence, sketch the graph of h. [3]

d(i). For the graph of h^{-1}, write down the x-intercept; [1]

d(ii). For the graph of h^{-1}, write down the equation of the vertical asymptote. [1]

e. Given that $h^{-1}(a) = 3$, find the value of a. [3]

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f.

a. Write down the value of q. [1]

b. The graph of f has a y-intercept at $(0,\ 4)$. [4]

Find the value of p.

c. The graph of f has a y-intercept at $(0,\ 4)$. [1]

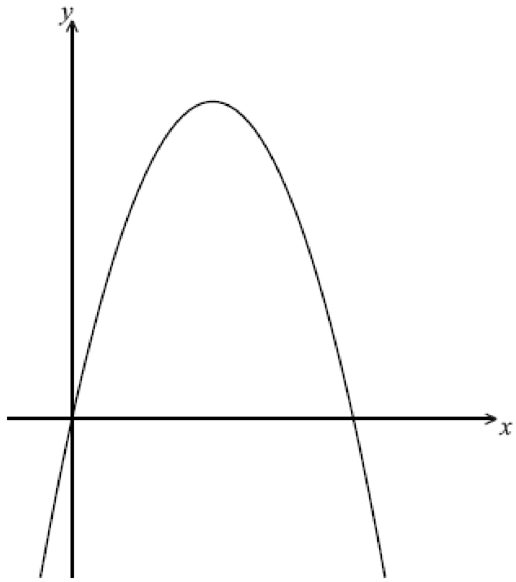
Write down the equation of the horizontal asymptote of the graph of f.

Let $f(x) = px^3 + px^2 + qx$.

a. Find $f'(x)$. [2]

b. Given that $f'(x) \geqslant 0$, show that $p^2 \leqslant 3pq$. [5]

Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



a. Find the x-intercepts of the graph. [4]

b(i) and (ii). (i) Write down the equation of the axis of symmetry. [3]

(ii) Find the y-coordinate of the vertex.

The velocity v ms^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

a. Write down the velocity of the particle when $t = 0$. [1]

b(i) and (ii). When $t = k$, the acceleration is zero. [8]

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

c. When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. [4]

Sketch a graph of v against t .

d(i) and (ii). Let d be the distance travelled by the particle for $0 \le t \le 1$. [3]

(i) Write down an expression for d .

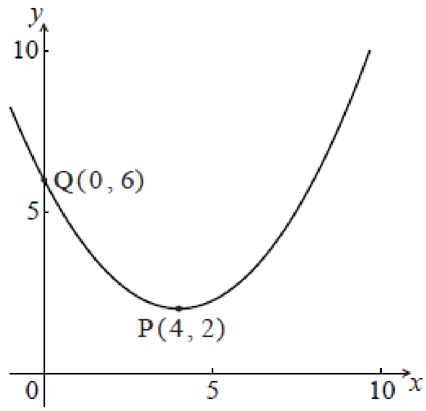
(ii) Represent d on your sketch.

Let $f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5$.

a. Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3]

b. Find the values of p so that $f(x) = 0$ has two **equal** roots. [3]

Let f be a quadratic function. Part of the graph of f is shown below.



The vertex is at P(4, 2) and the y-intercept is at Q(0, 6) .

a. Write down the equation of the axis of symmetry. [1]

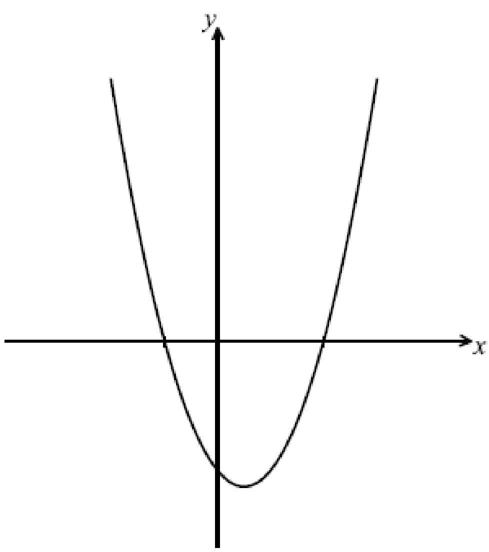
b. The function f can be written in the form $f(x) = a(x - h)^2 + k$. [2]

Write down the value of h and of k .

c. The function f can be written in the form $f(x) = a(x - h)^2 + k$. [3]

Find a .

The following diagram shows part of the graph of f , where $f(x) = x^2 - x - 2$.



a. Find both x-intercepts. [4]

b. Find the x-coordinate of the vertex. [2]

Consider $f(x) = \ln(x^4 + 1)$.

The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

a. Find the value of $f(0)$. [2]

b. Find the set of values of x for which f is increasing. [5]

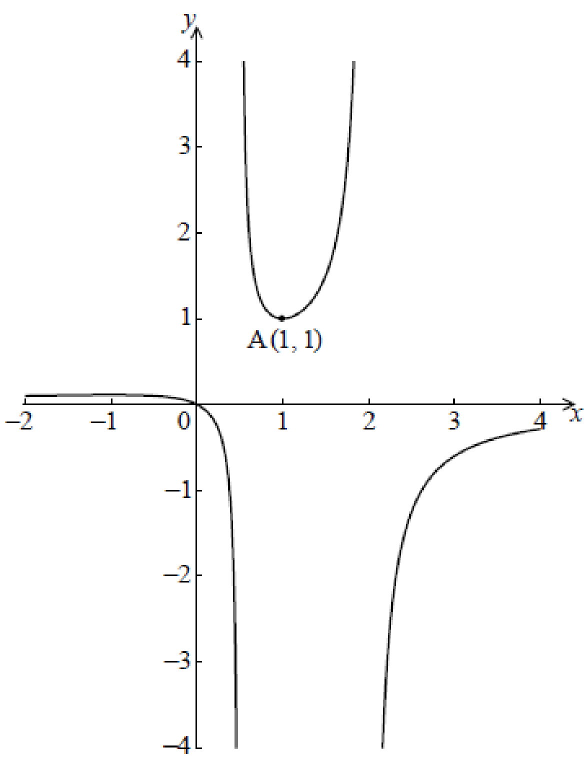
c. (i) Find $f''(1)$. [5]

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.

d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. [3]

Sketch the graph of f , for $x \geq 0$.

Let $f(x) = \frac{x}{-2x^2+5x-2}$ for $-2 \leq x \leq 4$, $x \neq \frac{1}{2}$, $x \neq 2$. The graph of f is given below.



The graph of f has a local minimum at A(1, 1) and a local maximum at B.

a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. [6]

b. Hence find the coordinates of B. [7]

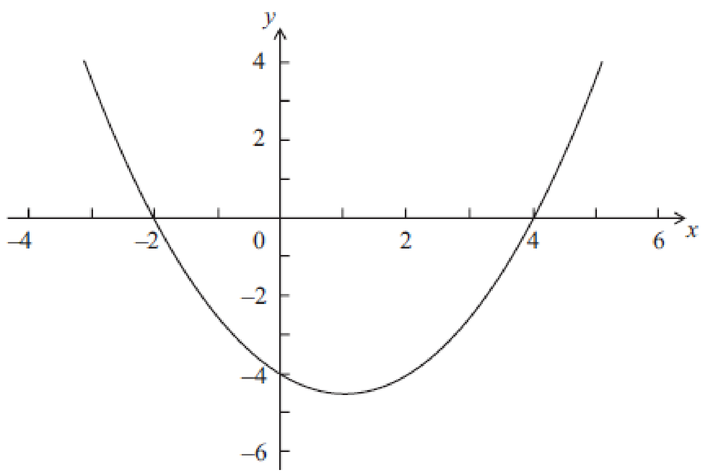
c. Given that the line $y = k$ does not meet the graph of f, find the possible values of k. [3]

---

a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. [2]

b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

Let $f(x) = p(x - q)(x - r)$. Part of the graph of f is shown below.



The graph passes through the points $(-2, 0)$, $(0, -4)$ and $(4, 0)$.

a. Write down the value of q and of r. [2]

b. Write down the **equation** of the axis of symmetry. [1]

c. Find the value of p. [3]

Let $f(x) = 6x\sqrt{1 - x^2}$, for $-1 \leqslant x \leqslant 1$, and $g(x) = \cos(x)$, for $0 \leqslant x \leqslant \pi$.

Let $h(x) = (f \circ g)(x)$.

a. Write $h(x)$ in the form $a\sin(bx)$, where $a,\ b \in \mathbb{Z}$. [5]

b. Hence find the range of h. [2]

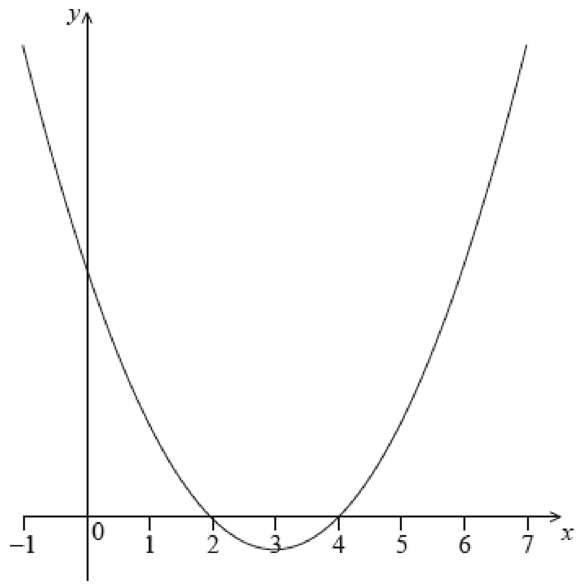
A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0, \ -6)$

a. Find the value of p. [3]

b. Find the value of a. [3]

c. The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(3, \ -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

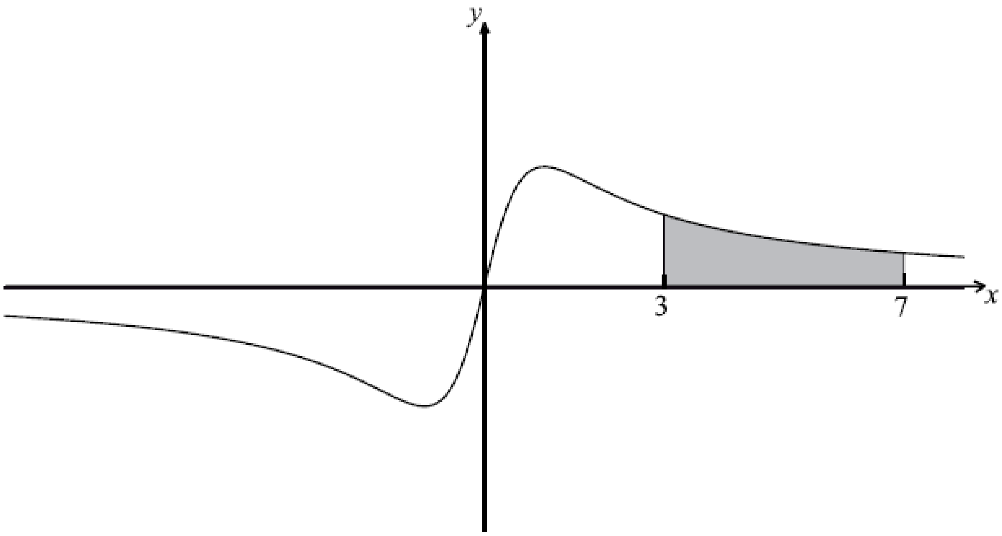
The function can also be written in the form $f(x) = (x - a)(x - b)$.

a. Write down the value of h and of k. [2]

b. Write down the value of a and of b. [2]

c. Find the y-intercept. [2]

Let $f(x) = \frac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$ .The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

a. Show that $f(-x) = -f(x)$. [2]

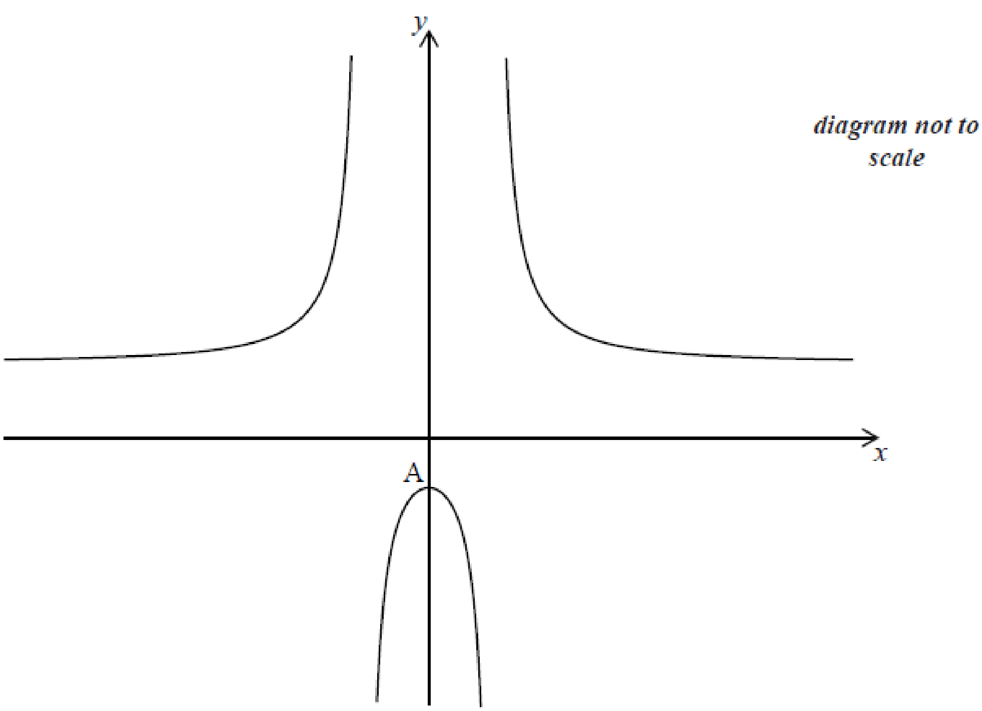
b. Given that $f''(x) = \frac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. [7]

c. It is given that $\int f(x)\mathrm{d}x = \frac{a}{2}\ln(x^2+1) + C$. [7]

(i) Find the area of the shaded region, giving your answer in the form $p\ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

Let $f(x) = 3 + \frac{20}{x^2-4}$, for $x \neq \pm 2$. The graph of f is given below.



The y-intercept is at the point A.

a. (i) Find the coordinates of A. [7]

(ii) Show that $f'(x) = 0$ at A.

b. The second derivative $f''(x) = \frac{40(3x^2+4)}{(x^2-4)^3}$. Use this to [6]

(i) justify that the graph of f has a local maximum at A;

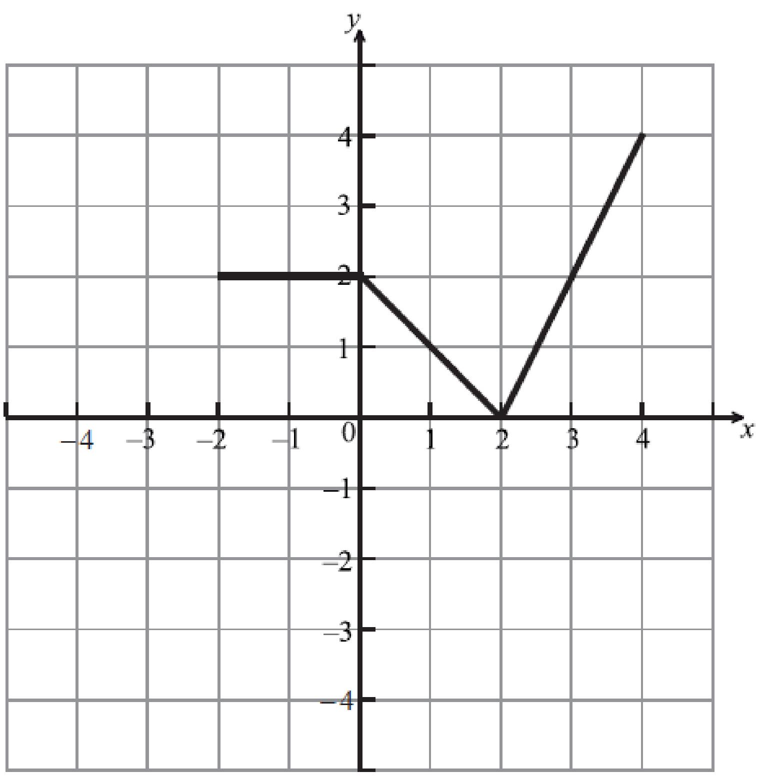
(ii) explain why the graph of f does **not** have a point of inflexion.

c. Describe the behaviour of the graph of f for large $|x|$. [1]

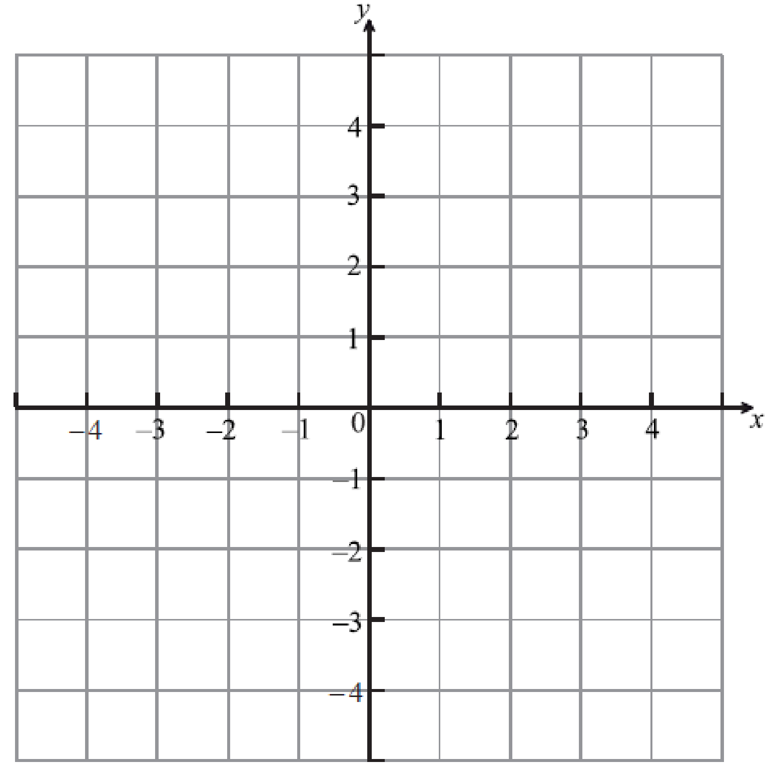
d. Write down the range of f . [2]

Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 4$.



a. Let $h(x) = f(-x)$. Sketch the graph of h on the grid below. [3]



b. Let $g(x) = \frac{1}{2}f(x-1)$. The point $\mathrm{A}(3, 2)$ on the graph of f is transformed to the point P on the graph of g . Find the coordinates of P. [3]

Let $f(x) = 3(x+1)^2 - 12$.

a. Show that $f(x) = 3x^2 + 6x - 9$. [2]

b(i), (ii), (iii) and (iv). For the graph of f [8]

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts.

c. **Hence** sketch the graph of f . [2]

d. Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations: [3]

a stretch of scale factor t in the y-direction

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t.

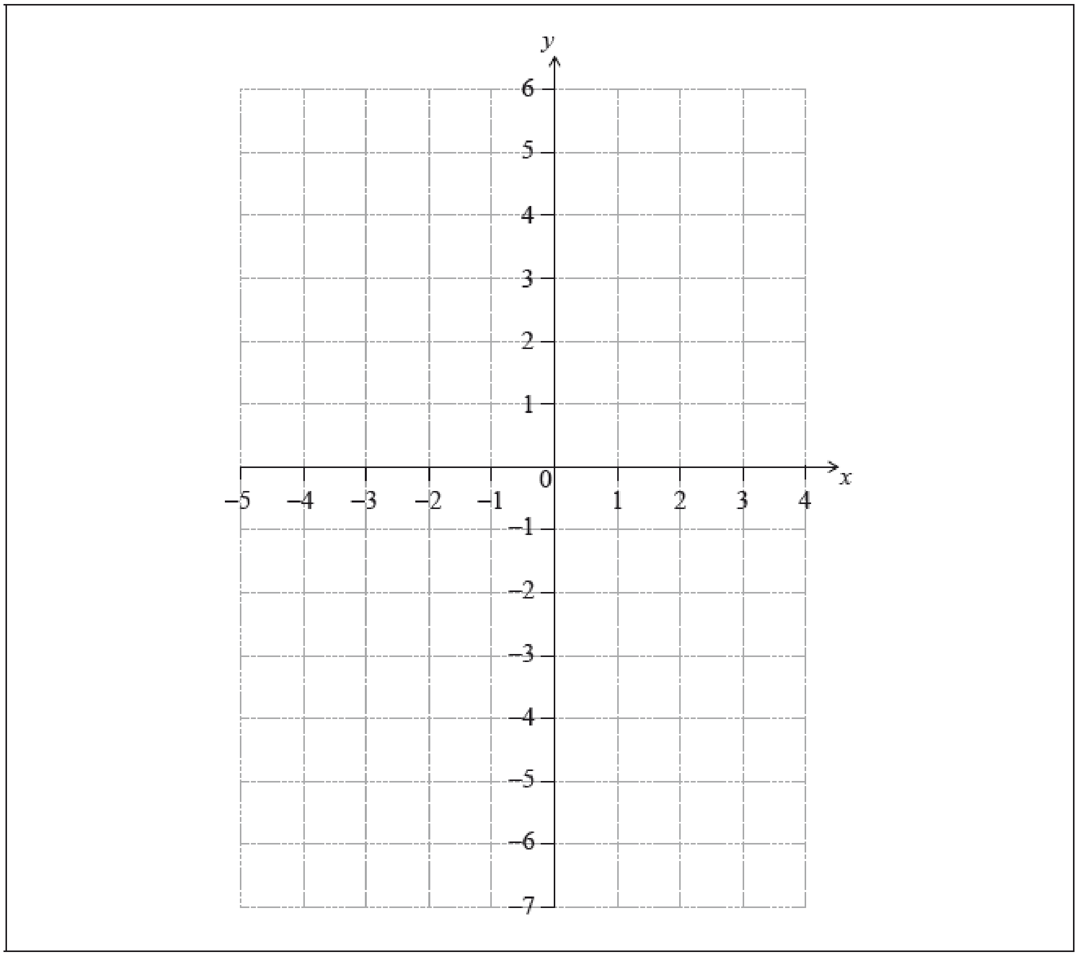
The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k .

Let $f(x) = x^2 + x - 6$.

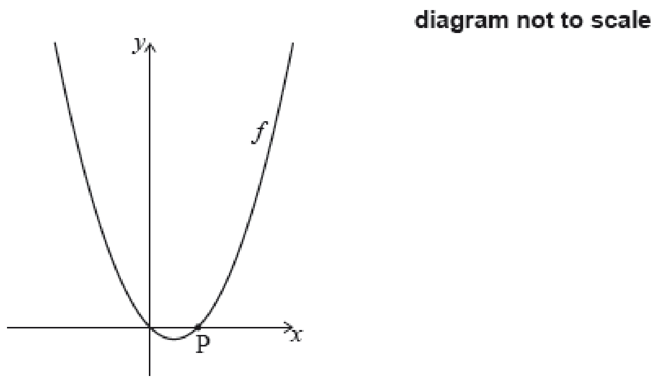
a. Write down the y-intercept of the graph of f. [1]

b. Solve $f(x) = 0$. [3]

c. On the following grid, sketch the graph of f, for $-4 \leq x \leq 3$. [3]



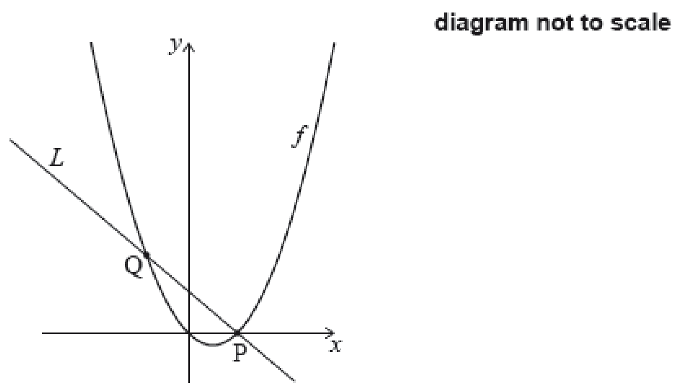
Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1,\ 0)$.

The line *L* is the normal to the graph of *f* at P.

The line L intersects the graph of f at another point Q, as shown in the following diagram.



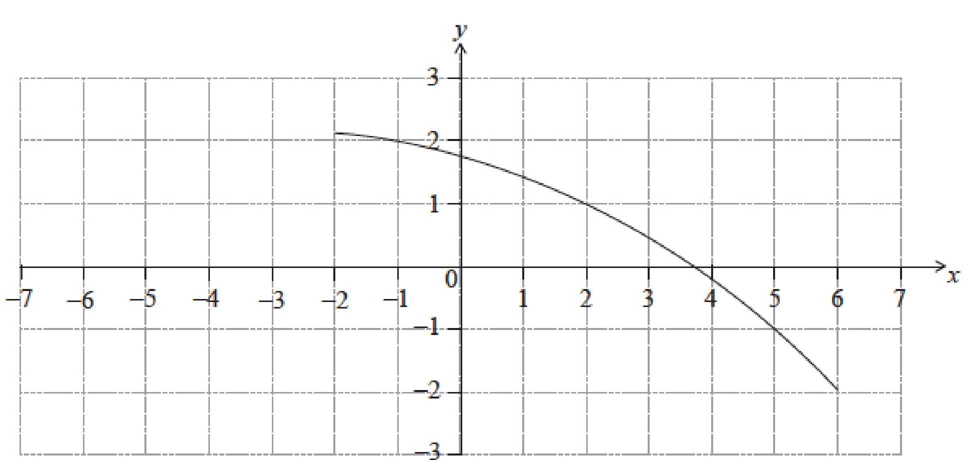
a. Show that $f'(1) = 1$. [3]

b. Find the equation of L in the form $y = ax + b$. [3]

c. Find the x-coordinate of Q. [4]

d. Find the area of the region enclosed by the graph of f and the line L. [6]

The following diagram shows the graph of a function f.



a. Find $f^{-1}(-1)$. [2]

b. Find $(f \circ f)(-1)$. [3]

c. On the same diagram, sketch the graph of $y = f(-x)$. [2]

---

a. Find the value of $\log_2 40 - \log_2 5$. [3]

b. Find the value of $8^{\log_2 5}$. [4]

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

a. Find $f^{-1}(2)$. [3]

b. Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$. [3]

The equation $x^2 + (k+2)x + 2k = 0$ has two distinct real roots.

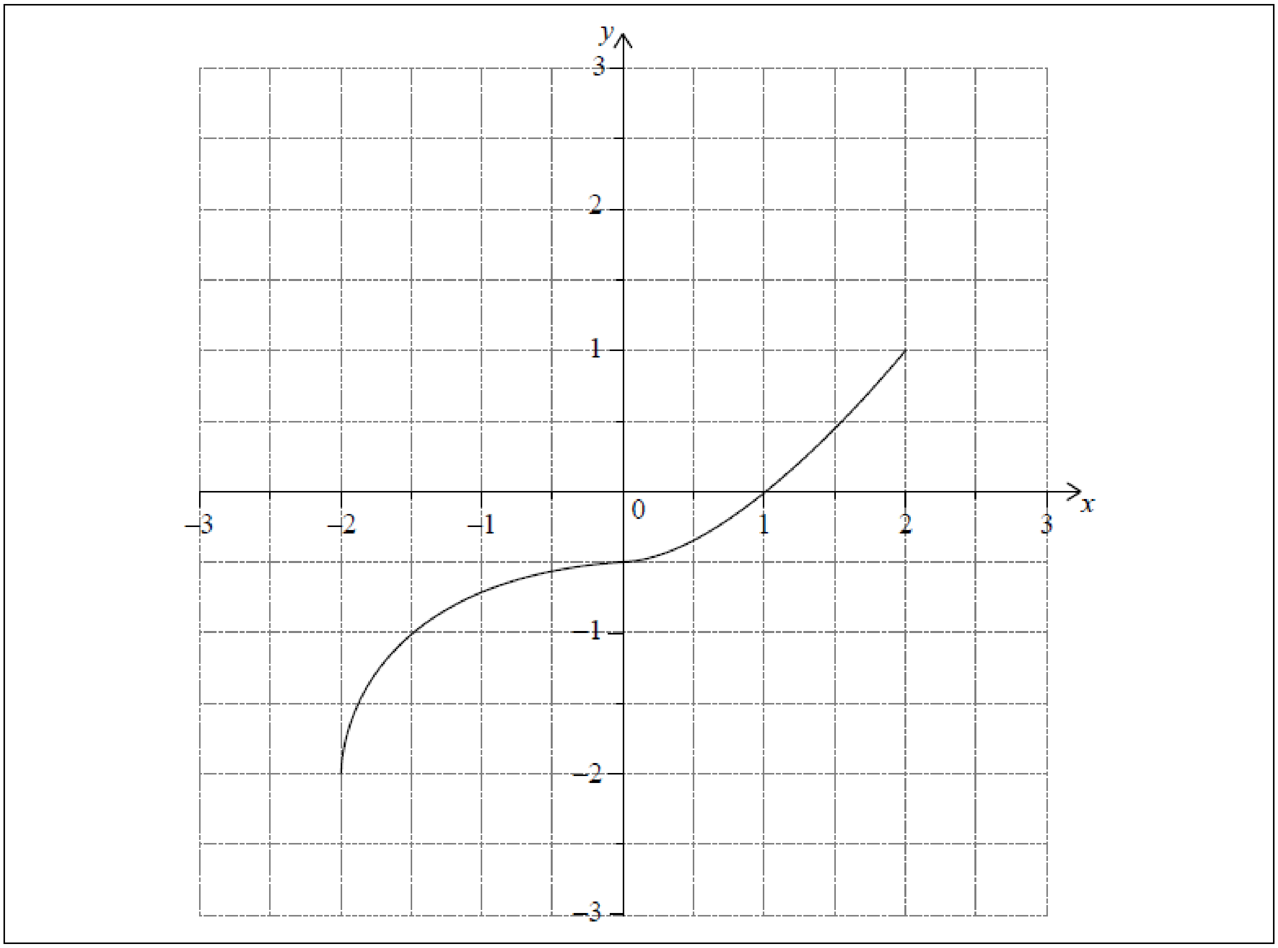
Find the possible values of k.

Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

a. Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4]

b. The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation. [3]

Consider a function *f*(*x*) , for −2 ≤ *x* ≤ 2 . The following diagram shows the graph of *f*.



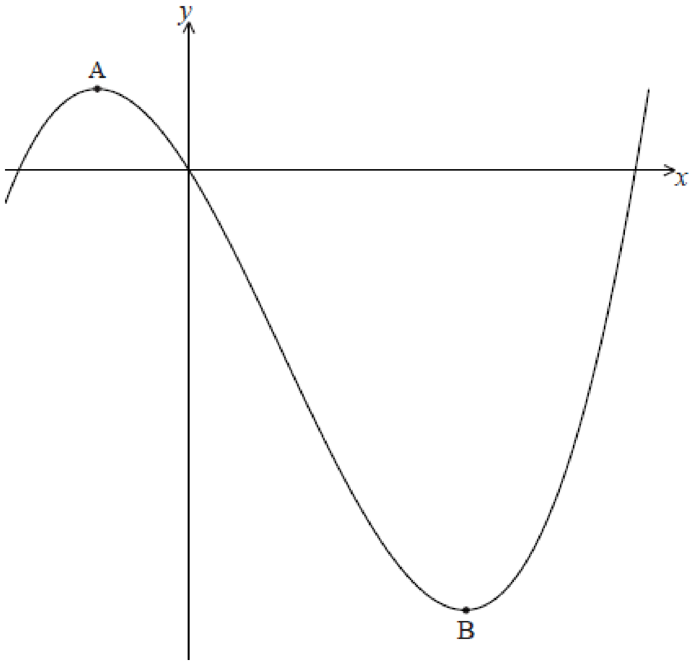
a.i. Write down the value of $f(0)$. [1]

a.ii. Write down the value of $f^{-1}(1)$. [1]

b. Write down the range of f^{-1}. [1]

c. On the grid above, sketch the graph of f^{-1}. [4]

Let $f(x) = \frac{1}{2}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B(3, − 9) .

a. Find the coordinates of A. [8]

b(i), (ii) and (iii). Write down the coordinates of [6]

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

The y-coordinate of P is $\ln 27$.

a. Find the x-coordinate of P. [3]

b. Find $f(x)$, expressing your answer as a single logarithm. [8]

c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$. [[N/A

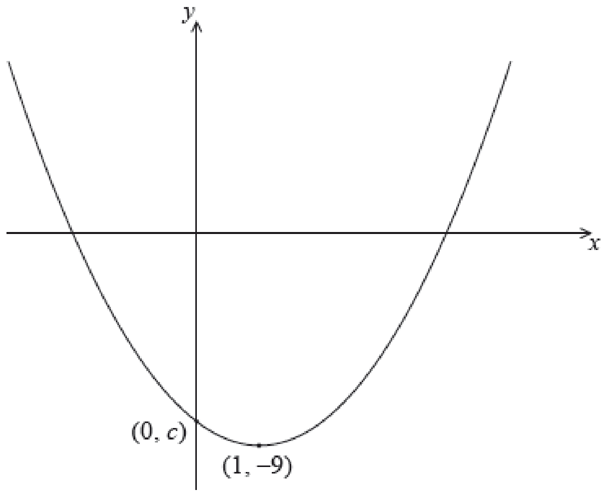
Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

a. Find the value of k . [5]

b. The line $y = p$ intersects the graph of f . Find all possible values of p . [2]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(1,\ -9)$, and the graph crosses the *y*-axis at the point $(0,\ c)$.

The function can be written in the form $f(x) = (x - h)^2 + k$.

a. Write down the value of h and of k. [2]

b. Find the value of c. [2]

c. Let $g(x) = -(x - 3)^2 + 1$. The graph of g is obtained by a reflection of the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$. [5]

Find the value of p and of q.

d. Find the *x*-coordinates of the points of intersection of the graphs of f and g. [7]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$P(A = a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta = \frac{3}{4}$. [6]

b. Given that $\tan\theta > 0$, find $\tan\theta$. [3]

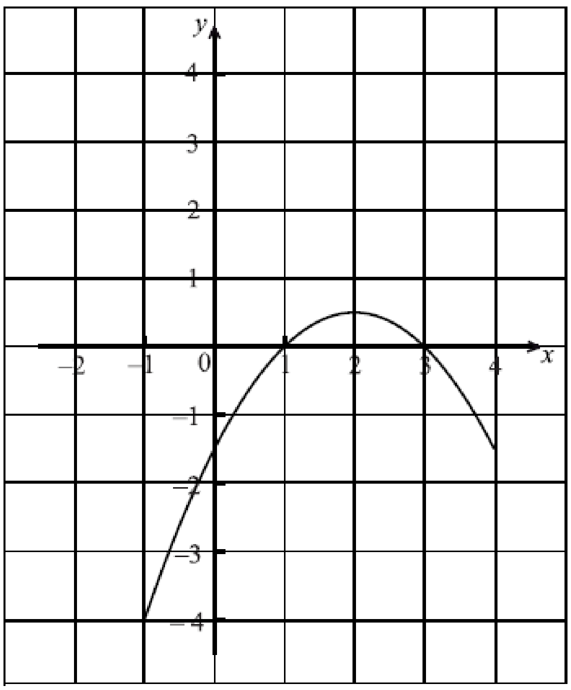
c. Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

a. Find $g^{-1}(x)$. [2]

b. Find $(f \circ g)(4)$. [3]

Part of the graph of a function *f* is shown in the diagram below.



a. On the same diagram sketch the graph of $y = -f(x)$. [2]

b(i) and (ii). Let $g(x) = f(x+3)$. [4]

(i) Find $g(-3)$.

(ii) Describe fully the transformation that maps the graph of *f* to the graph of *g*.

Consider the equation $x^2 + (k-1)x + 1 = 0$, where *k* is a real number.

Find the values of *k* for which the equation has two **equal** real solutions.

a. Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. [3]

b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. [3]

Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find Find the number of roots for the equation $f(x) = 0$.

Justify your answer.

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

a.i. The equation of the axis of symmetry is $x = p$. Find p. [2]

a.ii. Hence, show that $a = 2$. [2]

b. The equation of L is $y = 5$. Find the value of c. [3]

Let $f(x) = e^{x+3}$.

a. (i) Show that $f^{-1}(x) = \ln x - 3$. [3]

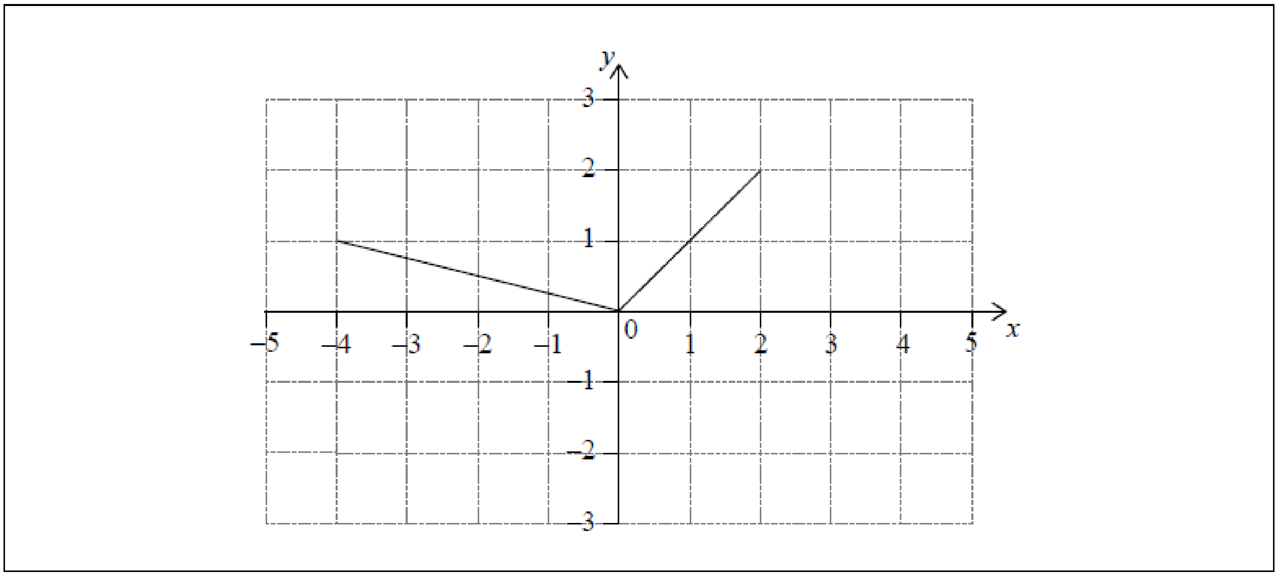
(ii) Write down the domain of f^{-1} .

b. Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. [4]

Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$.

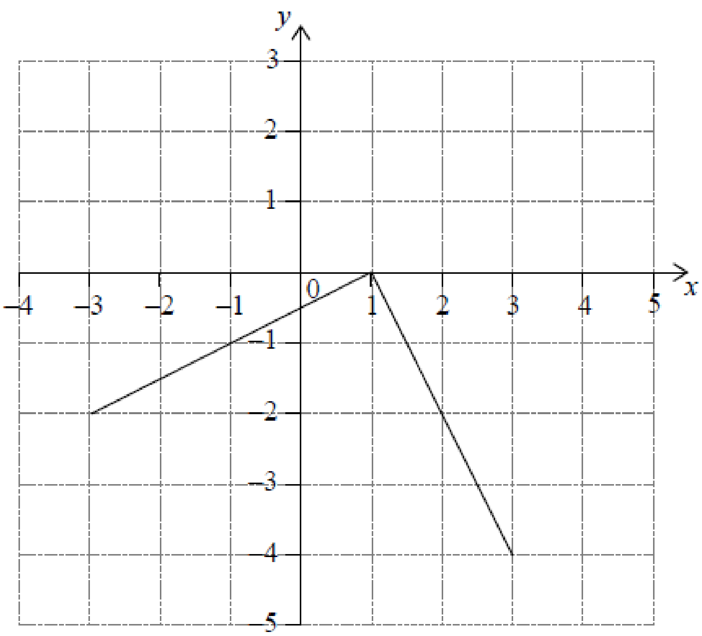
Find the possible values of x.

The following diagram shows the graph of a function f, for $-4 \leq x \leq 2$.



a. On the same axes, sketch the graph of $f(-x)$. [2]

b. Another function, g, can be written in the form $g(x) = a \times f(x+b)$. The following diagram shows the graph of g. [4]



Write down the value of *a* and of *b*.

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

The function g can be expressed in the form $g(x) = a(x-h)^2 + 3$.

a. Find $f'(x)$. [3]

b. Find $g(4)$. [3]

c. (i) Write down the value of h . [4]

(ii) Find the value of a .

d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . [6]

Write down the value of

a(i).(i) $\log_3 27$; [1]

a(ii).(ii) $\log_8 \frac{1}{8}$; [1]

a(iii).(iii) $\log_{16} 4$. [1]

b. Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. [3]

Let $f(x) = 3\tan^4 x + 2k$ and $g(x) = -\tan^4 x + 8k\tan^2 x + k$, for $0 \leqslant x \leqslant 1$, where $0 < k < 1$. The graphs of f and g intersect at exactly one point. Find the value of k.
